Methods for the optimal completion of modular rotors are proposed, which ensure a reduction in imbalances during the assembly phase, as well as noise and vibration levels during operation. The formulation of an optimization problem with various variants of the objective function and constraints is considered. Recommendations are given on the choice of the objective function option taking into account the specific operating conditions and the production of rotors. A form of technological documents is proposed to take into account the parameters of the modules and the optimal plan for the acquisition of rotors. The description of algorithms and the statistical data testifying the efficiency of optimization are resulted.

Keywords: noise, vibration, unbalance, rotor, module, assembly, optimization, target function, pair permutations, random variable.

Unbalanced rotors are common sources of increased noise levels and technological vibration as harmful and dangerous production factors [1], which are the cause of the decline in labor productivity, lack of comfort and professional pathology. In addition to the negative impact on people, vibration can destabilize the operation of various technical means and production processes, reduce their reliability, and also lead to accidents [2, 3].

To balance the rotors, different methods are used, the essence of which is to correct the unbalanced masses. This can be achieved by removing (adding) additional mass in the correction planes by drilling, grinding, soldering, welding or gluing. For modular type rotors, the designs of which consist of several modules, the balancing can be carried out by optimal picking without mass correction during the assembly phase. In this case, balancing is achieved due to mutual compensation of imbalances created by individual modules [4; 5].

The existing methods of optimal manning can be divided into two main groups. The first group includes methods of an experimental nature: the method of individual selection, the method of complete interchangeability, the method of group interchangeability (selective assembly), the method of ingroup interchangeability, and the method of acquisition with ranking parameters.

Common shortcomings of the second and third methods are incomplete production, caused by the difference between the laws of distribution of the parameters of elements, as well as the complexity of use in small-scale and single production. The first, fourth and fifth methods are also ineffective in small-scale and single-shot production due to the limited choice of elements.

The second group includes computational and analytical methods. Obviously, the task of completing the batch of rotors is optimized and refers to nonlinear discrete programming. Considering the fact that the rotor can include several modules (in some cases, more than ten), the problem is multidimensional, and can be solved using a computer. To complete a single flexible rotor, algorithms based on the individual selection method [7] can be used. Their disadvantage is the impossibility of simultaneous

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Methods of Modular Type Rotors Optimal Complexing
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Методи оптимального комплектування роторів модульного типу в процесі складання
acquisition of the entire batch of rotors, and the mathematical models used are not suitable for rigid rotors.

For the acquisition of a batch of rigid rotors from the corresponding batches of component elements (modules), a number of optimization methods [8—10], realized with the help of a computer can be applied. These include: the method of clipping (based on the Gomori algorithm), combinatorial methods (branch and bound method, dynamic programming methods and sequential optimization methods) and approximate methods. The first two groups are characterized by a relatively high complexity of algorithms and software implementation in the case of multidimensional problems.

Thus, the analysis of literary sources showed that all existing methods have certain shortcomings that limit their practical application. As a result, an effective reduction in the unbalances of modular rotors is still an urgent task today.

It is obvious that it is advisable to develop methods for the optimal assembly of modular rotors using computers that will minimize the imbalance of the rotors at the stage of their assembly without correcting the mass. This optimization assumes a 100% input control (measurement) of the parameters of the modules.

The purpose of the article is to prove the possibility of minimizing the imbalances in the batch of rigid rotors of the modular type by their optimal manning.

SETTING THE PROBLEM OF THE OPTIMAL SET OF THE TECHNOLOGICAL PARTY OF MODULAR TYPE ROTORS

The initial data in this problem are generally:
- the maximum permissible value of the specific imbalance of the manned rotor \( e_{\text{MAX}} \), \( \mu \text{m} \);
- maximum permissible value of the main moment unbalances module of the complete rotor \( M_{D,\text{MAX}} \), \( \text{g mm}^2 \);
- maximum permissible value of the target function \( F_{\text{MAX}} \);
- minimum allowable number of complete rotors \( N_{\text{MIN}} \);
- number of calculation cycles \( K \) (\( k \) — current cycle number);
- characteristics of modules (elements) arriving at the assembly (Table 3.1).

The mounting dimensions determine the position of each module in the common rotor coordinate system (in complete). The presence of private coordinate systems is caused by the need for separate manufacturing of modules (with the exception of co-processed products). The composition of the initial data is determined by the design features of the rotors and the requirements imposed on them. In specific cases, this composition can be adjusted.

The following variants of the objective function (CF) and constraints for single-purpose optimization are proposed, see Table 2. On the basis of these data, multipurpose optimization is also possible using one of the following methods: the use of a single functional, optimizing for several indicators, selecting a leading indicator, etc. However, due to the limited amount of work, multipurpose optimization is not considered in this paper.

As an example, we consider a simplified model of a modular rotor, shown in Fig. 1. Module number 1 is the base one. It is installed module № 2 and module № 3 with the possibility of various options for mutual location. In this case, the angles of module installation can take values of 0 or 180. Elements for modules fixing, as well as tolerances, landings and clearances are not considered to simplify the task.

The specific imbalance of the rotor (\( \mu \text{m} \)) is

\[
e_i = 10^6 \sqrt{\left(\sum_{j=1}^{m} M_j l_j \cos [\arccos(a_{ij}) + \varphi_j] \right)^2 + \left(\sum_{j=1}^{m} M_j h_j \sin [\arcsin(h_{ij}) + \varphi_j] \right)^2},
\]

where \( i \) — ordinal number of the rotor in the process lot; \( n \) — number of rotors; \( j \) — module (element) type number in the rotor; \( m \) — number of module types; \( M_j \) — module weight; \( h_j \) and \( l_j \) — coordinates (m) of the centers of mass of the modules, measured in particular coordinate systems along the axes \( OY \) and \( OX \), respectively (see Fig. 1); \( \varphi_j \) — angle (rad.) of the module base surface rotation in the common rotor coordinate system (angle of module installation in the rotor).

The main moment modulus (\( \text{g mm}^2 \)) of imbalances is

\[
M_{D,i} = 10^6 \sqrt{\left(\sum_{j=1}^{m} M_j l_j (Z_i - Z_{ij}) \cos [\arccos(a_{ij}) + \varphi_j] \right)^2 + \left(\sum_{j=1}^{m} M_j h_j (Z_i - Z_{ij}) \sin [\arcsin(h_{ij}) + \varphi_j] \right)^2},
\]

where \( Z_i \) — coordinate (m) of the rotor center of mass along the axis \( OZ \) after assembly.

| Table 1. Characteristics of modules (elements) of the j-type |
| Таблица 1. Характеристики модулів (елементів) j-го типу |

<table>
<thead>
<tr>
<th>Module number (i) in the process container before assembly</th>
<th>Weight of the module ( M_j, \text{g} )</th>
<th>Coordinates of the module’s center of mass in a private system ( X''Y''Z'' ), mm</th>
<th>Module mounting dimensions in the common rotor coordinate system ( XYZ ), mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1...n )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
For a given rotor design, the coordinates of the centers of mass of the module (element) 2 ($C_1$) and module 3 ($C_2$) in the $XYZ$ system are calculated, respectively, according to the formulas:

$$Z_1 = \sum_{j=1}^{n} M_{yj} Z_{yj},$$

$$Z_2 = \sum_{j=1}^{n} M_{yj} Z_{yj}.$$  

In each variant of Table 2, $CF$ is an integral quality indicator, quantitatively characterizing the process of acquisition of a batch of rotors. In the first version, the $DF$ represents the arithmetic mean of the specific imbalances, and in the second variant the arithmetic mean of the modules of the main instants of the imbalances of the entire batch of rotors. In the first two options, these should be kept to a minimum. In the third version, the $CF$ characterizes the presence of work in progress. In general, the choice of the $DF$ can be performed taking into account the design features of attaching the rotor to the operation site, experimental data on noise and vibration, and technological features of production. Therefore, it is suggested that the $FT$ is selected depending on the priority of the problem caused by the imbalance of the rotor. If a greater noise level is observed with a static imbalance than with a momentary imbalance, then variant 1 should be chosen as the $FT$ (see Table 2). For experimental evaluation of noise and vibration levels, reference rotors with reference masses in the correction planes can be used.

Output data in the general case are:

- the value of the objective function $F$;
- specific imbalances $e$ for each rotor;
- module of the main moment of imbalances $M_D$ for each rotor;
- the number of complete rotors $N$ satisfying the constraints.

**Table 2. Suggested options for $CF$ and restrictions**

<table>
<thead>
<tr>
<th>Variant number</th>
<th>Target function</th>
<th>Restrictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$F_1 = \frac{1}{n} \sum_{i=1}^{n} e_i \rightarrow \min$</td>
<td>$e_i \leq e_{\text{MAX}}$, $M_{D1} \leq M_{\text{MAX}}$, $N \geq N_{\text{MIN}}$</td>
</tr>
<tr>
<td>2</td>
<td>$F_2 = \frac{1}{n} \sum_{i=1}^{n} M_{D1} \rightarrow \min$</td>
<td>$F_2 \leq F_{\text{MAX}}$</td>
</tr>
<tr>
<td>3</td>
<td>$F_3 = N \rightarrow \max$</td>
<td>$F_3 \geq N_{\text{MIN}}$</td>
</tr>
</tbody>
</table>

**Fig. 1. Sketch of the modular type rotor drawing**

Рис. 1. Ескіз креслення ротора модульного типу

**Fig. 2. Algorithm for optimal acquisition of rotors on the basis of pair permutations of modules**

(аналогічна функція відповідає варіанті 1, див. табл. 2)
The optimal plan for manning (see Table 3).

The technological document for assembling the batch of rotors (assembly sheet, picking card) is proposed in the form shown in Table 3.

To solve this problem, two methods are proposed, listed below.

METHOD AND ALGORITHM OF OPTIMAL COMPLETION BASED ON STEAL PERMANENTLY

This method is designed to optimally fit the rotors with the only possible value for the installation angles of modules relative to the reference surface (key). The method is based on a heuristic search with pair permutations of incoming elements (modules), and can be implemented using a computer [11]. The algorithm for calculating the optimal picking plan is shown in Fig. 2.

Block 1 organizes the formation of an initial picking plan, which can be a random combination of modules or approximated to the optimal combination as a result of preliminary optimization by one of the known methods. In block 6, the basic is the rotor (string) under the number. The remaining rotors are current. In blocks 9 and 11, unsuccessful permutations are counted.

If all permutations between the selected rotors (lines) do not lead to an improvement in the result (block 11), the next current rotor is moved (block 12). The calculation is repeated many times (the number of calculation cycles is — K). At the end of the calculation, the best result is displayed. The convergence of this algorithm (in probability) to the global optimum is obvious.

METHOD AND ALGORITHM OF OPTIMAL COMPLETION BASED ON RANDOM SEARCH

This method is based on the Monte Carlo method [9; 10]. The block diagram of the algorithm is shown in Fig. 3. Block 2 generates random uniformly distributed values for the installation angles of modules and their numbers from the corresponding process lots (see Table 1). In block 3, the values of the objective function and constraints are calculated in accordance with the selected option (see Table 3.2). In blocks 4—7, the relevant restrictions are checked.

---

**Table 3. Output data of optimization calculation**

<table>
<thead>
<tr>
<th>Rotor number after assembly</th>
<th>Specific imbalance ( e_j, \mu m )</th>
<th>The main imbalances ( M_{d,j}, g \times mm^2 )</th>
<th>Number of comp. rotors ( N )</th>
<th>Target function value ( F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1...n</td>
<td>/</td>
<td>/</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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**Figure 3. Algorithm for optimal acquisition of rotors based on the Monte Carlo method**

Рис. 3. Алгоритм оптимального комплектування роторів на основі методу Монте-Карло

Block 8 evaluates the result of the current calculation cycle by comparing it with the result of the previous cycle. The best option is retained and is basic for subsequent cycles. The result of the calculation is the best option obtained when all the calculation cycles are completed.

As the number of calculation cycles increases, the probability of achieving a global optimum increases, which is calculated from formula (4).

---

**Figure 4 — Dependence of the probability of achieving the global optimum from \( K \) (Mathcad [12])**

Рисунок 4 — Залежність ймовірності досягнення глобального оптимуму від \( K \) (Mathcad [12])
The scientific novelty consists in setting up an optimization task, recommendations on the choice of the target function, proposed methods and algorithms for solving, and also in obtaining statistical data on the convergence of the result to a global optimum.

**Practical significance** consists in confirming the possibility of minimizing the imbalances in a batch of rigid rotors of the modular type by their optimal manning without correcting the mass, the need for 100% input control of the module parameters, and also in using the proposed forms of technological documents. The materials of this article can be used to organize the technological processes of assembling the rotors of this type, which will significantly reduce their imbalances, reduce noise and vibration, and increase reliability during operation.

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